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ATOMIC COHERENT STATE FOR A SYSTEM OF THREE-LEVEL ATOMS

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Atomic coherent states for an ensemble of three-level atoms are defined. Some their properties and possible applications are discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Когерентное состояние системы трехуровневых атомов

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Построены атомные когерентные состояния системы трехуровневых атомов, обсуждаются некоторые их свойства и возможные их практические применения.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Many problems in quantum optics can be dealt with in terms of the interaction of an ensemble of three-level atoms with a transverse electromagnetic field. The model consisting of an ensemble of N identical three-level atoms driven by resonant external fields was used for describing the collective behaviour of a double resonance /1/, resonant Raman scattering /2-5/, superfluorescence /6-8/, four-wave mixing /12/. On the analogy of the atomic coherent states for the ensemble of two-level atoms /9-11/ the cogerent states for the ensemble of three-level atoms will be defined in this paper. Their properties and possible applications are discussed too.

For the case when the effect of a different spatial position of atoms is ingnored, the ensemble of N three-level atoms can be characterized by collective angular momentum operators as follows: $J_{ij} = \sum_{k=1}^{N} |i\rangle_k\langle j|_k$ $(i, j = 1,2,3)$.

They obey the commutation relations

$$[J_{ij}, J_{i'j'}] = J_{ij'}\delta_{ji'} - J_{i'j}\delta_{ij'}$$

we introduce the eigenstates of the operators J_{11}, $J_{11}+J_{22}$ and operator of the total number of atoms $\hat{N} = J_{11} + J_{22} + J_{33}$

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$$\hat{N}|P,Q> = N|P,Q>$$
$$J_{11}|P,Q> = Q|P,Q>$$
$$(J_{11} + J_{22})|P,Q> = P|P,Q>, \qquad (1)$$
where $0 \le P \le N$, $0 \le Q \le P$.
It is easy to show that:

$$J_{12}^{m}|P,Q> = [\frac{(P-Q)!}{(P-Q-m)!} \cdot \frac{(Q+m)!}{Q!}]^{1/2}|P,Q+m> \qquad (2)$$

$$J_{21}^{n}|P,Q> = [\frac{(P-Q+n)!}{(P-Q)!} \cdot \frac{Q!}{(Q-n)!}]^{1/2}|P,Q-n> \qquad (3)$$

$$J_{32}^{\ell}|P,Q> = [\frac{(N-P+\ell)!}{(N-P)!} \cdot \frac{(P-Q)!}{(P-Q-\ell)!}]^{1/2}|P-\ell,Q> \qquad (4)$$

$$J_{23}^{k}|P,Q> = [\frac{(N-P)!}{(N-P-k)!} \cdot \frac{(P-Q+k)!}{(P-Q)!}]^{1/2}|P+k,Q>. \qquad (5)$$

Analogously to the coherent states for the system of two-level atoms/9/ we introduce the coherent states for the system of three-level atoms in the form:

$$|\mu,\beta> = A^{-1/2} e^{\beta J_{12}} e^{\mu J_{23}} |0,0>$$
$$= A^{-1/2} \sum_{P=0}^{N} (\frac{N!}{(N-P)!P!})^{1/2} \mu^{P} \sum_{Q=0}^{P} (\frac{P!}{(P-Q)!Q!})^{1/2} \beta^{Q}|P,Q>, \qquad (6)$$

where μ and β run over the complex plane and A is a normalization factor. We have

$$<\beta,\mu|\mu,\beta> = A^{-1}(1+|\mu|^2+|\mu|^2|\beta|^2)^N$$

and hence the normalization factor has the form

$$A(|\mu|,|\beta|) = (1+|\mu|^2+|\mu|^2|\beta|^2)^N. \qquad (7)$$

The overlap integral between two states $|\mu_1,\beta_1>$ and $|\mu_2,\beta_2>$ is

$$<\beta_1,\mu_1|\mu_2\beta_2> = A^{-1/2}(|\mu_1|,|\beta_1|)A^{-1/2}(|\mu_2|,|\beta_2|)(1+\mu_1^*\mu_2(1+\beta_1^*\beta_2))^N \qquad (8)$$

whence

$$|<\beta_1,\mu_1|\mu_2,\beta_2>|^2 = A^{-1}(|\mu_1|,|\beta_1|)\cdot A^{-1}(|\mu_2|,|\beta_2|)|1+\mu_1^*\mu_2+\mu_1^*\mu_2\beta_1^*\beta_2|^{2N}.$$

From definition (6) one can find

$$(\beta J_{12} - J_{11})|\mu,\beta> = 0$$
$$(\frac{1}{\beta}J_{21} - J_{22})|\mu,\beta> = 0 \qquad (9)$$
$$(\frac{1}{\mu}J_{32} - J_{33})|\mu,\beta> = 0.$$

These equations, together with

$$\hat{N} |\mu, \beta> = N |\mu, \beta>$$

specify uniquely the coherent states. The coherent states $|\mu,\beta>$ form minimum-uncertainty packets. The uncertainty relation can be defined, for example, in terms of the set of operators

$$J_x = \frac{1}{2}(J_{21} + J_{12}), \quad J_y = \frac{i}{2}(J_{12} - J_{21}), \quad J_z = \frac{1}{2}(J_{22} - J_{11}).$$

These three observables obey a commutation relation

$$[J_x, J_y] = i J_z$$

whence they have the uncertainty property

$$(\Delta J_x)^2 (\Delta J_y)^2 \geq \frac{1}{4} (\Delta J_z)^2.$$

It is easy to show that the equality sign holds for the coherent state $|\mu,\beta>$ that is therefore a minimum-uncertainty state.

Let us now consider the completeness properties of the coherent states (6). Using the relation

$$\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = \frac{(m-1)!(n-1)!}{(m+n-1)!}$$

and the completeness of states $|P,Q>$ one obtains

$$\frac{(N+2)(N+1)}{\Pi^2} \int d^2\mu \, d^2\beta \frac{|\mu|^2}{(1+|\mu|^2+|\mu|^2|\beta|^2)^3} |\mu,\beta><\beta,\mu| \tag{10}$$
$$= \sum_{P=0}^{N} \sum_{Q=0}^{P} |P,Q><Q,P| = 1.$$

Then, the expansion of an arbitrary state $|P,Q>$ foolows

$$|P,Q> = \frac{(N+2)(N+1)}{\Pi^2} \left(\frac{N!}{(N-P)!(P-Q)!\,Q!}\right)^{1/2}.$$

$$\int d^2\mu \, d^2\beta \frac{|\mu|^2 \mu^{*P} \beta^{*Q}}{(1+|\mu|^2+|\mu|^2|\beta|^2)^{\frac{N}{2}+3}} |\mu,\beta>. \tag{11}$$

Thus, analogously to the coherent spin state /10/ and atomic coherent state for two-level atoms /9,11/, we define the atomic coherent state for the system of three-level atoms. The coherent atomic states $|\mu,\beta>$ possess a number of properties: (i) The states are defined by a unitary transformation operator $e^{\beta J_{12}} e^{\mu J_{23}}$ acting on the ground state; (ii) the states obey simple eigenvalue equations;

(iii) these states are nonorthogonal and overcomplete;
(iv) minimum-uncertainty relations for noncommuting operators can be constructed within the atomic coherent states. It is easy to see that the defined atomic coherent states (6) are suitable to describe the resonant interaction of a set of three-level atoms with a classical fields. In the atomic coherent state the master equation produces a number of differential equations. Take, for example, the problem of resonant Ramant scattering /5/ (see the figure). Such a system is described by the reduced atomic density operators that obey the master equation /5/

$$\frac{\partial \rho}{\partial t} = -i\Omega[\cos\alpha\,(J_{12}+J_{21}) + \sin\alpha\,(J_{23}+J_{32}), \rho] + \gamma_{21}(J_{12}\rho J_{21} - \rho J_{21}J_{12} + \text{H.C.}) + \gamma_{23}(J_{32}\rho J_{23} - \rho J_{23}J_{32} + \text{H.C.}), \tag{12}$$

where $2\gamma_{21}$ and $2\gamma_{23}$ are radiative spontaneous transition probabilities per unit time for a single atom to change from the level $|2\rangle$ to $|1\rangle$ and from $|2\rangle$ to $|3\rangle$, respectively; $\Omega = (\Omega_1^2 + \Omega_2^2)^{1/2}$ and $\mathrm{tg}\,\alpha = \Omega_2/\Omega_1$. Here Ω_1 and Ω_2 are the Rabi frequencies for the atomic transitions from the level $|2\rangle$ to $|1\rangle$ and from $|2\rangle$ to $|3\rangle$, respectively.

It is easy to show that in the atomic coherent state the master equation (12) reduces to

$$\frac{\partial \rho(\mu^*,\beta^*,\mu,\beta,\tau)}{\partial \tau} = \Big\{\Big(\mu^*\beta^*\frac{\partial}{\partial\mu^*} - \beta^{*2}\frac{\partial}{\partial\beta^*} - \frac{\partial}{\partial\beta}\Big)\cdot\Big[-i\cos\alpha + g_1\Big(\mu\beta\frac{\partial}{\partial\mu} - \beta^2\frac{\partial}{\partial\beta}\Big)\Big] + \Big(\frac{\partial}{\partial\mu^*} - \frac{\beta^*}{\mu^*}\frac{\partial}{\partial\beta^*} - N\mu + \mu^2\frac{\partial}{\partial\mu}\Big)\cdot\Big[-i\sin\alpha + g_2\Big(\frac{\partial}{\partial\mu} - \frac{\beta}{\mu}\frac{\partial}{\partial\beta}\Big)\Big]\Big\}\rho(\mu^*,\beta^*,\mu,\beta,\tau) + \text{H.C.}, \tag{13}$$

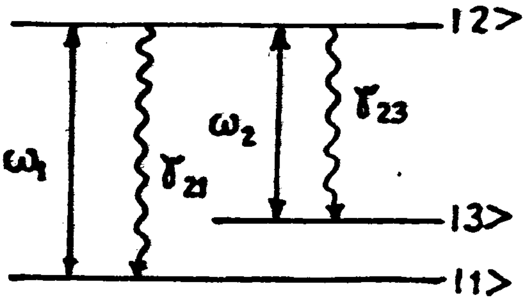

Fig. Schematic representation of three-level system interacting with resonant incident and scattered coherent waves.

where

$\rho(\mu^*, \beta^*, \mu, \beta, \tau) = \langle \beta, \mu | \rho | \mu, \beta \rangle \quad A(|\mu|, |\beta|)$

$\tau = \Omega t$ and $g_1 = \frac{\gamma_{21}}{\Omega}$, $g_2 = \frac{\gamma_{21}}{\Omega}$.

Equation (13) can be considered as the Fokker-Planck equation for the system of three-level atoms in the resonant driving fields.

The investigation of equation (13) and the corresponding cooperative properties of resonant Raman scattering will be represented in other publications.

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